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Professor Carslaw has given us, probably, the best balanced elementary book on non-euclidean geometry, both its history and its theory, which has been written; for that he deserves our sincere gratitude. For the future, let us hope that, aided by further coöperation of men so instinct with life and enthusiasm as Liebmann, he can continue to enrich the mathematical world.

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A History of Elementary Mathematics, with Hints on Methods of Teaching. By FLORIAN CAJORI, Ph.D. Revised and Enlarged Edition. The Macmillan Company, New York and London, 1917. viii + 324 pages. \$1.75.

A general demand for information in regard to the historical development of all the great sciences, coupled with the questions that have recently been raised as to the advantage of omitting a considerable part of the present teaching of mathematics, makes Professor Cajori's *History of Elementary Mathematics* a timely and welcome addition to the libraries of both teachers and students. In accordance with the title, the emphasis of the book is upon the foundations of the science of mathematics. Some advanced theories are mentioned, but puzzling details are avoided, much to the relief of the general reader, who knows that many of the upper regions in mathematics can be reached only by the climbing of stairways which are invisible until one has acquired some Aladdin's lamp of knowledge.

The historical proof of the development of mathematics as a firm and progressive science, rather than as a changing mental plaything or business calculating machine, is suggested at once in the table of contents. This short and philosophical summary furnishes to the teacher of mathematics, or to the general reader of history, the material from antiquity to modern times which enables him to place the subject where it belongs—among the old and reliable and constantly developing sciences. At practically every point which is taken up, quite full and satisfactory references are given, so that authorities on past and present historical material are placed before the reader. Those who are familiar with Professor Cajori's *History of Mathematics*, dated December, 1893, will recognize with pleasure the reappearance of many of the well-preserved mathematical antiques which he then used to exhibit clearly and forcibly the contributions of the science during the ages. The authorship and the construction of many of these antiques are again shown to be insolvable problems, but the discussion of the possible originators forms an interesting and valuable part of the new book.

Among racial contributions the book presents with great appreciation the work of the Hindus and the Arabs, and mentions the recently discovered numeral records of the Mayas of Central America as an early attempt on the part of

American races to develop mathematics; but discussion of the possibility of more important discoveries and contributions by the Japanese and Chinese than have so far been credited to them is omitted. The index includes a fairly large number of names of early mathematicians and present-day writers, but those who desire to look up such a connection as that between logic and mathematics find no assistance in the index, though the subject is discussed on p. 289; and there is also no reference to the treatment on p. 300 of the introduction of graphic methods. All these items are of interest to the teacher of the present day and should be included in the index. One wishes also that the question that is quoted so often and in so many kinds of writing as a mathematical problem of the middle ages, "How many angels can stand on the point of a needle?", had been traced to its source, in order to save the time of teachers and students in history, philosophy, literature or mathematics, who are anxious to know its origin.

The tendencies and work of the contributors to mathematics are brought out in a satisfactory way. All the old and famous mathematicians are recognized with appropriate commendations. Where space for details is limited, the lives and qualities and mathematical work of the contributors are presented in a vivid manner by a brief phrase, or even a single adjective, and make the book very readable, even to those who have usually regarded mathematics as a dry subject. Fibonacci is described as "a business man whose leisure hours were given to mathematical study," and Archimedes, "while admired by his fellow citizens for his mechanical inventions," "himself prized more highly his discoveries in pure science." The mathematician who invented a useless new method, more for a pastime than for a practical short cut, is exhibited with a proper reprimand; and the Greek mathematicians, who failed to do their share in geometry because they allowed no construction except by ruler and compasses, receive their due criticism for placing style before progress.

The lack of advance in geometry from the time of the ancient Greeks until Desargues and Pascal in the seventeenth century, and the way their discoveries were "neglected until the close of the eighteenth century" have been carefully noted, as well as the astounding fact that there was a period of "two thousand years when Egyptian mathematics was stationary." These and other points call attention to the curious delays in developments in periods when ideas were ready to flower, but were not cultivated by their owners or observed by more watchful eyes. Over and over again Professor Cajori brings out the lack of advance in theory in time to meet new and necessary demands, because attention was paid so often to arbitrary rules rather than to principles. His presentation of unsatisfactory attitudes toward arithmetic, especially in the 17th century, in the earlier part of the volume, affords considerable material for those readers who wish to take up the question outlined in his quotation from Abraham Flexner, "If, for example, only so much arithmetic is taught as people actually have occasion to use, the subject will sink into modest proportions."

The general tendency of most writers on mathematics who wish to reach modern readers is constantly to use modern notations and methods in the

elementary operations, but in this book comparisons of past and present methods are made possible by the use of ancient styles in discussing some of the fundamental processes. This enables the author to emphasize the pressure of principles as a factor in leading to new discoveries and improved notation, even though the forms in which these principles were first presented pass away, as in his summary of the difficulties due to the present notation in dealing with limits and irrationals. It seems possible that the remark of Professor Cajori, p. 236, "In case of radicals, what a gain it would be, if we could burst the chains which tie us to the past," may furnish the pressure which is needed to do away with many handicaps in notation, especially for the introduction of the metric system to complete our decimal furniture, or the addition of graphic representation in elementary mathematics.

All through the volume the influence of the needs of practical life is emphasized, and case after case is given which shows how great advances in theory have come from those peoples, and from those periods, wherein the demand led solitary mathematicians and busy teachers to give extra time and thought to methods of training the next generation and to seeking help of every sort. The biological sources are indicated when the quinary, decimal, and duodecimal number systems are called "the three systems based on human anatomy." The fields where late cultivation has brought out new results are illustrated by notice of the modern work on triangles. The way in which long and unfruitful efforts to solve a practical problem have finally led to the addition of a previously unknown number characteristic is shown in the study of the history of the quadrature of the circle. Through such discussions Professor Cajori opposes the attitude of certain philosophers who "would make us believe that observation is not needed or developed in mathematical study," and helps the average person to realize the fact that even a young mathematical student may bring to light a peculiarly valuable short cut or make a worthy discovery.

On the whole, the book contains an amount of material and suggestions that is likely to lead many readers to agree with the conclusion which Professor Cajori states in his last sentence, "Certain it is that the teaching of mathematics will require constant modifications in order to keep pace with the ever changing needs of modern life."

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